JOURNAL OF AIRCRAFT
Vol. 35, No. 5, September–October 1998

Frequency Effects on Dynamic Stability Derivatives Obtained from Small-Amplitude Oscillatory Testing

Douglas I. Greenwell*
Defence Evaluation and Research Agency, Clapham, Bedfordshire MK41 6AE, England, United Kingdom

As a result of a continuing program of work to establish a basic understanding of the aerodynamic phenomena that influence agility, stability, and control of future combat aircraft configurations, it has become clear that the conventional stability or aerodynamic derivative model for the representation of aerodynamic loads in the aircraft equations of motion is no longer adequate. This paper discusses the limitations of the stability derivative model for modern combat aircraft maneuvers, in particular the problem of motion frequency effects in static and dynamic derivatives derived from small-amplitude oscillatory wind-tunnel tests, and presents an alternative modeling technique based on the concept of an aerodynamic transfer function.

Nomenclature

A = in-phase component of rolling moment response
AR = amplitude ratio
a_i, b_i, c_i = constants in generalized transfer functions
B = in-quadrature component of rolling moment response
b = wingspan, m
C_l = rolling moment coefficient, L/qSb
$C_{l\beta,\text{att}}$ = attached flow component of steady-state derivative $C_{l\beta,0}$
$C_{l\beta}$, $C_{l\beta,0}$ = steady-state static rolling moment derivative, $=\partial C_l/\partial\beta$, rad^{-1}
$C_{l\dot\beta}$, $C_{l\dot\beta,0}$ = steady-state dynamic rolling moment derivative, $\partial C_l/\partial(\dot\beta b/2U)$, rad^{-1}
$C_{l\beta,\text{sep}}$ = separated flow component of steady-state derivative $C_{l\beta,0}$
$C_{l\beta,\Omega}$ = frequency dependent static rolling moment derivative
$C_{l\dot\beta,\Omega}$ = frequency dependent dynamic rolling moment derivative
c = aerodynamic mean chord, m
c_r = root chord, m
f = frequency, Hz
$G(s)$ = transfer function
k = reduced frequency of plunging motion, $\pi f c_r/U$
k_H = constant derived from integration of unit step response
L, M, N = rolling, pitching, and yawing moments, Nm
m, n = order of transfer function
p, q, r = rotational velocity components, rad/s
q = dynamic pressure, $1/2\rho U^2$, Pa
S = reference wing area, m^2
s = Laplace transform variable
$\bar{s}$ = nondimensional Laplace transform variable, $sb/2U$
T = time constant, s
t = time, s
U = freestream velocity, m/s
u, v, w = translational velocity components, m/s
X, Y, Z = axial, side, and normal forces, N
x, y, z = Cartesian coordinates, m
y_0 = amplitude of sinusoidal sway motion
α = angle of attack, $\tan^{-1}(w/u)$
β = sideslip angle, $\sin^{-1}(v/U)$
β_0 = amplitude of sinusoidal sideslip motion
ρ = density of air, kg/m^3
T = nondimensional time constant
τ = nondimensional time, $2Ut/b$
ϕ = phase lag
Ω, Ω_b = nondimensional lateral frequency, $\omega b/2U$
Ω_n = nondimensional natural frequency of rolling moment response
ω = frequency, rad/s

Introduction

THE increasing maneuver capabilities of modern combat aircraft have highlighted the shortcomings of the conventional stability or aerodynamic derivative-based model for the representation of aerodynamic loads in the aircraft equations of motion. Of particular concern is the presence of significant motion frequency effects on the static and acceleration derivatives measured in small-amplitude oscillatory wind-tunnel tests at higher angles of attack that cannot be reconciled with the stability derivative model.

Although these effects were first recognized in the 1950s, they have not in general been significant for conventional aircraft maneuvering at relatively low rates and amplitudes. However, the stability derivative model breaks down completely for modern combat aircraft with highly nonlinear aerodynamic characteristics undergoing agile maneuvers at high angles of attack. It is now widely acknowledged that it is essential to improve the modeling of the nonlinear, time-dependent aerodynamic responses during these maneuvers, to maximize combat capability and prevent accidental departure from controlled flight.

A number of nonlinear modeling techniques are now under investigation, including nonlinear indicial response (U.S. and Canada), Fourier functional analysis (U.S.), aerodynamic transfer functions (France), and state-space representation (Russia, France, and U.S.). However, practical applications have been limited because of the mathematical complexity of the modeling processes and the limited capabilities of existing dynamic wind-tunnel test facilities.

This paper discusses the limitations of the stability derivative model for modern combat aircraft maneuvers, in particular the problem of motion frequency effects in static and dynamic derivatives derived from small-amplitude oscillatory wind-tunnel tests, and it presents an alternative modeling technique based on the concept of an aerodynamic transfer function.

Received July 13, 1997; revision received Feb. 24, 1998; accepted for publication Feb. 27, 1998.

*Principal Scientist, High Speed and Weapons Aerodynamics.

Aerodynamic Derivative

The concept of the aerodynamic or stability derivative, introduced by Bryan[1] in the early days of aviation, remains essentially unchanged as the conventional method for the representation of aerodynamic loads in the equations of motion of an aircraft. However, despite the success of this method, it is by no means mathematically sound.[2] Indeed, it does not give correct results even for linear aerodynamic characteristics for cases where the aerodynamic forces change rapidly.

The method of Bryan is based on the assumption that the aerodynamic forces and moments are a function of the instantaneous values of the disturbance velocities and control angles and their derivatives. Expressions for the forces and moments are then obtained in the form of a Taylor series expansion in these variables, linearized by discarding the higher-order terms.

Taking the rolling moment acting on an aircraft during a pure sideslipping motion as an example, we have the two conventional nondimensional aerodynamic derivatives

$$C_{l\beta} = \frac{\partial C_l}{\partial \beta}, \qquad C_{l\dot\beta} = \frac{\partial C_l}{\partial\left(\dfrac{\dot\beta b}{2U}\right)} \tag{1}$$

giving the instantaneous rolling moment

$$C_l(t) = C_{l\beta}\,\beta(t) + C_{l\dot\beta}\frac{\dot\beta(t) b}{2U} \tag{2}$$

However, the definition of the truncated Taylor series expansion in Eq. (2) is mathematically flawed, in that β and $\dot\beta$ are not independent variables.

An alternative approach to deriving the aerodynamic derivatives is given by Hancock,[3] who considers the linear response of the aerodynamic forces and moments acting on an aircraft to a general time variation of the incidence angles as the sum of a sequence of incremental step responses. For the rolling moment response this approach gives

$$C_l(t) = C_{l\beta}\,\beta(t) - \{C_{l\beta} k_H + [\text{residual term}(t)]\}[\dot\beta(t)b/2U] \tag{3}$$

where k_H is a constant derived from integration of the unit aerodynamic step response function and the time-dependent residual term is a function of the form of $\beta(t)$. Equation (3) highlights two points:

1) There are no higher order terms, i.e., $C_{l\ddot\beta}$, etc., do not exist.

2) The conventional model of Eqs. (1) and (2) only applies if the residual term is small, in which case $C_{l\dot\beta} \approx -C_{l\beta} k_H$

For quasisteady motions, low-frequency undamped oscillations, and exponential divergent motions the residual term is typically less than 10% of $C_{l\beta} k_H$ and may safely be ignored.[3] Unfortunately, for the damped high-rate motions typical of combat aircraft maneuvers this is not the case, and the simple derivative model breaks down, even for linear aerodynamic characteristics.

The derivatives due to the translational velocities α ($\approx w/u$) and β ($\approx v/U$) are determined from steady-state wind-tunnel experiments and, hence, are often referred to as static derivatives. For example

$$C_{l\beta} \approx \Delta C_l / \Delta\beta \tag{4}$$

where $\Delta\beta$ is typically ± 5 deg. The choice of $\Delta\beta$ is a compromise between generating sufficiently large moments to measure accurately, and the often highly nonlinear static aerodynamic characteristics at intermediate angles of attack. Note that the use of two incidence angles α and β rather than three velocity components u, v, and w implicitly restricts this modeling approach to flight conditions where the aircraft velocity U is constant or varying slowly. The derivatives resulting from p, q, and r are also, strictly speaking, aerodynamically steady state, but are often referred to (erroneously, and somewhat confusingly) as dynamic or damping derivatives because they are generally measured using a rotating rig of one form or another. These derivatives can also be obtained from fixed-axis oscillatory tests, but in combination with the translational derivatives.

The derivatives due to rates of change of velocity, i.e., $\dot\alpha$, $\dot\beta$, $\dot p$, $\dot q$, and $\dot r$, are determined from oscillatory wind-tunnel experiments and hence are conventionally referred to as dynamic or acceleration derivatives. Consider an aircraft model oscillated sinusoidally from side to side, i.e., a swaying motion, at a small amplitude

$$y(t) = -y_0 \cos(\omega t) \tag{5a}$$

which in terms of sideslip angle gives

$$\beta(t) \approx \dot y(t)/U = \beta_0 \sin(\omega t), \qquad \dot\beta(t) \approx \ddot y(t)/U = \beta_0 \omega \cos(\omega t) \tag{5b}$$

and, hence

$$\beta_0 = y_0 \omega / U \tag{5c}$$

The limitation to small amplitudes enables one to assume that the rolling moment response to this motion is linear,[4] hence

$$C_l(t) = A \sin(\omega t) + B \cos(\omega t) \tag{6}$$

The in-phase and in-quadrature components A and B are then determined from measured rolling moment time histories (allowing for wind-off tares caused by inertia forces) by one of a number of digital or analogue signal-processing techniques. From Eqs. (1) and (6), the rolling moment aerodynamic derivatives are

$$C_{l\beta} = \frac{\partial C_l}{\partial \beta} = \frac{A}{\beta_0}, \qquad C_{l\dot\beta} = \frac{\partial C_l}{\partial\left(\dfrac{\dot\beta b}{2U}\right)} = \frac{B}{\beta_0\left(\dfrac{\omega b}{2U}\right)} = \frac{B}{\beta_0 \Omega} \tag{7}$$

For quasisteady motions at low incidences, the static derivatives are generally sufficient to model the resultant aerodynamic loads. However, at higher incidences a number of studies[5,6] have shown that inclusion of the dynamic derivatives in the aerodynamic model can have a significant effect on calculated aircraft stability characteristics.

Unfortunately, the practical application of dynamic stability derivatives in aircraft flight dynamics simulations has caused a number of problems, primarily because of significant inconsistencies between oscillatory and steady-state data for a given configuration. These discrepancies stem from two root causes: 1) differences in experimental conditions and 2) variation of static and dynamic derivatives with oscillation frequency.

The first encompasses a wide range of factors, because oscillatory and steady-state tests are often undertaken on different model support systems, with different models, with different motion amplitudes, in different wind tunnels, and at different Reynolds numbers. Of particular concern for combat aircraft configurations are changes in support interference (because of flow sensitivity to downstream blockage), and in differences between motion amplitude and the changes in incidence angles over which the steady-state derivatives are determined (because of nonlinearity of the aerodynamic characteristics).

The second was first highlighted in the 1950s, as a result of a number of studies in the U.S. that showed that at higher

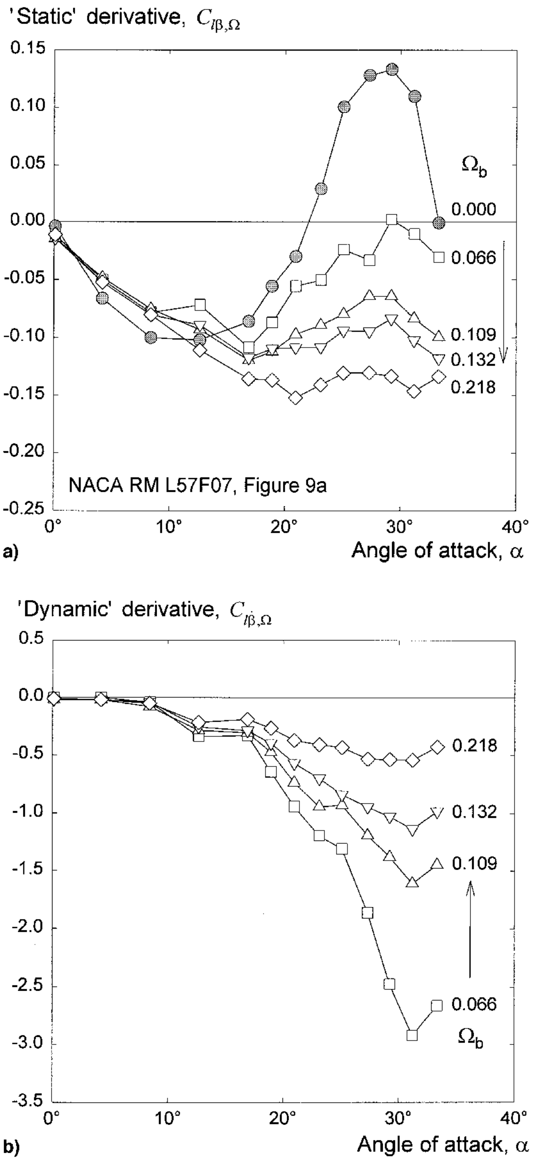

Fig. 1 Effect of frequency of oscillation on lateral derivatives for a 60-deg delta wing.

incidences the effects of frequency of oscillation on lateral derivatives could be very significant.[7-13]

Figure 1 illustrates this with data from Ref. 7 for a 60-deg delta wing (note that in this case the rolling moment is measured in stability rather than body axes). Typically, as frequency is reduced toward zero the static derivative tends toward its steady-state value, whereas the dynamic derivative increases continuously in magnitude.

The question raised by Fig. 1 is how to incorporate the frequency effects in a mathematical model of the time-dependent aerodynamic characteristics. The conventional aerodynamic derivative model provides no such means, and the resulting question of which value to use for the dynamic derivative has generally been dealt with in a rather ad hoc manner. Commonly, the steady-state values for the static derivatives are utilized, combined with values for the dynamic derivatives at a frequency thought to be most representative of the expected aircraft motions. Clearly, this is most unsatisfactory and it is now widely recognized that an alternative approach is required.

Aerodynamic Transfer Functions for Small-Amplitude Oscillatory Data

The frequency effects on both static and dynamic derivatives noted earlier can be clarified if the aerodynamic response to

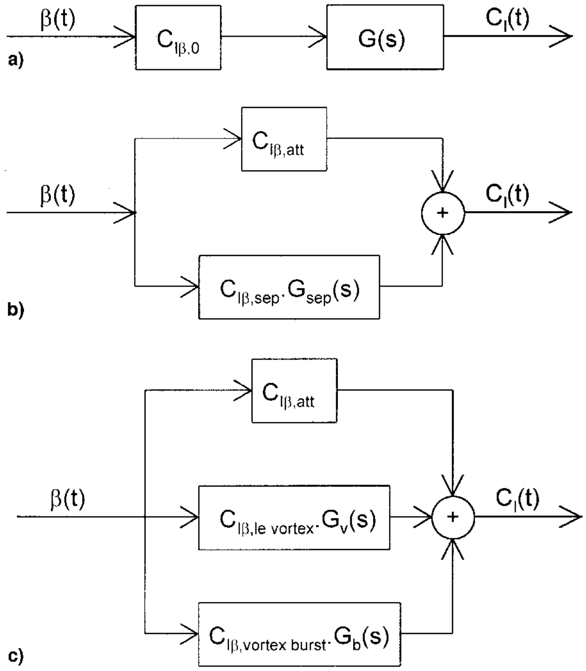

Fig. 2 System representations of time-dependent aerodynamics characteristics: a) simple model, b) attached and separated flow model, and c) leading-edge vortex flow model.

oscillatory aircraft motion is considered in terms of the response of a linear physical system to a sinusoidal input,[14] (Fig. 2a). The assumption of linearity is justified as before on the basis of the amplitude of the oscillatory input being small. The concept of what is effectively an aerodynamic transfer function (or ATF) is not new, being originally put forward by Etkin,[15] but appears to have been neglected in recent years in favor of more complex and less physically intuitive approaches, e.g. nonlinear indicial response,[16] Fourier functional analysis,[17] neural networks,[18] etc.

Continuing with the previous example of the rolling moment response to sideslip, let the input to the system shown in Fig. 2a be

$$\beta(t) = \beta_0 \sin(\omega t)$$

and the output

$$C_l(t) = (C_{l\beta,0}\beta_0)AR \sin(\omega t + \phi) \tag{8}$$

where AR and ϕ are the frequency-dependent amplitude ratio and phase angle of the system transfer function $G(s)$, respectively, and $C_{l\beta,0}$ is the steady-state rolling moment because of sideslip gain. From Eqs. (6–8), the frequency-dependent aerodynamic derivatives as measured from conventional small-amplitude oscillatory tests become

$$\begin{aligned} C_{l\beta,\omega} &= \frac{C_l}{\beta}(\omega t = \pi/2) = C_{l\beta,0}AR\cos(\phi) \\ C_{l\dot{\beta},\omega} &= \frac{C_l}{\dot{\beta}b/2U}(\omega t = 0) = \frac{C_{l\beta,0}AR\sin(\phi)}{\Omega} \end{aligned} \tag{9}$$

Now consider the behavior of Eq. (9) when ω becomes zero, where AR is 1.0, and ϕ is 0 deg, so that

$$\begin{aligned} C_{l\beta,0} &= C_{l\beta,0}(1.0)\cos(0°) = C_{l\beta,0} \\ C_{l\dot{\beta},0} &= \frac{C_{l\beta,0}(1.0)\sin(0°)}{(0b/2U)} = \frac{0}{0} \end{aligned} \tag{10}$$

giving no surprises for $C_{l\beta,0}$ but an indeterminate value for $C_{l\dot{\beta},0}$. Applying L'Hôpital's rule to Eq. (9) gives

$$\lim_{\omega \to 0} C_{l\dot{\beta},\omega} = C_{l\beta,0}\left[\frac{\sin(\phi)\dfrac{d(AR)}{d\omega} + AR\dfrac{d(\sin\phi)}{d\omega}}{\dfrac{d\Omega}{d\omega}}\right]_{\omega=0}$$

$$= C_{l\beta,0}\left[\frac{\cos(\phi)\dfrac{d\phi}{d\omega}}{(b/2U}\right]_{\omega=0} \tag{11}$$

For a damped linear physical system ϕ is the sum of the phase lags of the constituent first-order elements

$$\phi_{\text{total}} = \sum_{i=1}^{n}[-\tan^{-1}(\omega T_i)] \tag{12a}$$

which for small ϕ and ω becomes

$$\phi_{\text{total}} \approx \sum_{i=1}^{n}(-\omega T_i) = \omega\sum_{i=1}^{n}(-T_i) = -\omega T_{\text{eff}} \tag{12b}$$

where T_{eff} is the sum of the system time constants. Equation (12a) can be applied to transfer functions with lead terms (zeros) by changing the sign of the relevant phase lag contribution, i.e., to $+\tan^{-1}(\omega T_i)$. Substituting Eq. (12b) into Eq. (11) gives

$$\lim_{\omega \to 0} C_{l\dot{\beta},\omega} = C_{l\beta,0}\frac{-T_{\text{eff}}}{b/2U}$$

and, hence

$$C_{l\dot{\beta},0} = -\mathrm{T}_{\text{eff}}C_{l\beta,0} \tag{13}$$

where the nondimensional system time constant T_{eff} can be seen to be equivalent to the constant k_H in Eq. (3).

Equation (13) immediately shows that the dynamic derivative variation with frequency found experimentally is consistent with a damped linear system response, with the derivative tending to a nonzero steady-state value that is directly proportional to (but generally of opposite sign to) the steady-state value for the static derivative.

Application to Existing Experimental Data

To demonstrate the application of the aerodynamic transfer function concept to existing experimental data using a physically significant structure for the model, we once again use the example of rolling moment response to sideslipping motion. Although for the purposes of system identification the ATF concept should ideally be applied directly to the force and moment time histories, available oscillatory data are presented in the conventional derivative form. For the purposes of this discussion, we shall therefore continue to work in these terms.

A clue to a reasonable form for the system model is given by Ref. 19, where $C_{l\dot{\beta},\omega}$ was successfully estimated for a twin-jet fighter aircraft from experimental data for the steady-state and oscillatory values of $C_{l\beta,\omega}$ and a theoretical estimate for $C_{l\beta,0}$ in the absence of flow separations ($\equiv C_{l\beta,\text{att}}$). Using consistent notation, the response equation can be written as

$$C_{l\dot{\beta},\omega} = (C_{l\beta,\text{att}} - C_{l\beta,0})^{\sin\phi/\Omega} \tag{14}$$

where

$$\cos\phi = \left(\frac{C_{l\beta,\text{att}} - C_{l\beta,\omega}}{C_{l\beta,\text{att}} - C_{l\beta,0}}\right) \tag{15}$$

The derivation of this equation is given in full in Ref. 19, but to summarize, the rolling moment has effectively been split into two constituent parts: 1) an attached flow contribution with no time lag, and 2) a separated flow contribution that lags the sideslip angle by a phase angle ϕ.

The magnitudes of both are assumed to be constant for a given angle of attack, whereas ϕ is a function of Ω. The dynamic derivative is then simply the out-of-phase component of the separated flow contribution. A similar partition was applied in Ref. 20, for the composite yawing moment derivative ($C_{nr,\omega} - C_{n\dot{\beta},\omega}\cos\alpha$).

An equivalent approach can be taken in constructing a composite aerodynamic transfer function for the roll moment response, as shown in Fig. 2b. This has a reasonable degree of physical significance and also neatly avoids the difficulties caused by zero crossings in the steady-state derivative data (Fig. 1a), where the amplitude ratio of a simple transfer function would become infinite.

Working in nondimensional frequency Ω and time τ, the rolling moment response equation [Eq. (8)] becomes

$$\begin{aligned}\beta(\tau) &= \beta_0\sin(\Omega\tau)\\ C_l(\tau) &= (C_{l\beta,\text{att}}\beta_0)\sin(\Omega\tau)\\ &\quad + (C_{l\beta,\text{sep}}\beta_0)AR_{\text{sep}}\sin(\Omega\tau + \phi_{\text{sep}})\end{aligned} \tag{16}$$

where $C_{l\beta,\text{att}}$ and $C_{l\beta,\text{sep}}$ are the steady-state rolling moment gain contributions at a given angle of attack as a result of the attached and separated flow regions, respectively, whereas AR_{sep} and ϕ_{sep} are the frequency-dependent amplitude ratio and phase lag functions, respectively, for the separated flow response $G_{\text{sep}}(s)$.

Using Eqs. (8) and (9), Eq. (16) can be rewritten in terms of the conventional (frequency-dependent) static and dynamic derivatives as

$$\begin{aligned}C_{l\beta,\Omega} &= C_{l\beta,\text{att}} + C_{l\beta,\text{sep}}AR_{\text{sep}}\cos(\phi_{\text{sep}})\\ C_{l\dot{\beta},\Omega} &= \frac{C_{l\beta,\text{sep}}AR_{\text{sep}}\sin(\phi_{\text{sep}})}{\Omega}\end{aligned} \tag{17}$$

which can then be rearranged to give

$$C_{l\dot{\beta},\Omega} = [\tan(\phi_{\text{sep}})/\Omega](C_{l\beta,\Omega} - C_{l\beta,\text{att}}) \tag{18a}$$

$$C_{l\beta,\text{sep}} = C_{l\beta,0} - C_{l\beta,\text{att}} \tag{18b}$$

The frequency response of the separated flow aerodynamic transfer function $G_{\text{sep}}(s)$ can be determined by rearranging Eqs. (17) and (18) in terms of AR_{sep} and ϕ_{sep}

$$\begin{aligned}AR_{\text{sep}} &= \frac{\sqrt{(\Omega C_{l\dot{\beta},\Omega})^2 + (C_{l\beta,\Omega} - C_{l\beta,\text{att}})^2}}{(C_{l\beta,0} - C_{l\beta,\text{att}})}\\ \phi_{\text{sep}} &= \tan^{-1}\left(\frac{\Omega C_{l\dot{\beta},\omega}}{C_{l\beta,\Omega} - C_{l\beta,\text{att}}}\right)\end{aligned} \tag{19}$$

and plotting as a Bode or polar plot.

As a first step, Eqs. (18) were applied to experimental rolling moment data for a 60-deg delta wing.[7] This dataset was chosen for the single reason that it was the only study identified in the open literature with a sufficient frequency range (five points, $\Omega = 0.0$–0.218) to provide any hope of identifying a response model. Although there were considerable experimental inconsistencies between the steady-state and oscillatory tests, the data were felt to be adequate for the purposes of indicating the potential of the aerodynamic transfer function approach.

If we initially assume that the separated flow response can be approximated by a single first-order lag, the overall transfer function corresponding to Eq. (16) can be written in terms of nondimensional parameters as

$$G_{\mathrm{att}}(\bar{s}) = C_{l\beta,\mathrm{att}} + C_{l\beta,\mathrm{sep}}G_{\mathrm{sep}}(\bar{s})$$
$$= C_{l\beta,\mathrm{att}} + \frac{C_{l\beta,\mathrm{sep}}}{1 + \mathrm{T}_{\mathrm{sep}}\bar{s}} \tag{20a}$$

From Eq. (12), the phase lag of the separated flow contribution $G_{\mathrm{sep}}(s)$ is

$$\tan(\phi_{\mathrm{sep}}) = -\mathrm{T}_{\mathrm{sep}}\Omega \tag{20b}$$

and, hence, Eq. (18a) becomes

$$C_{l\dot{\beta},\Omega} = -\mathrm{T}_{\mathrm{sep}}(C_{l\beta,\Omega} - C_{l\beta,\mathrm{att}}) \tag{21}$$

Thus a cross plot of $C_{l\dot{\beta},\Omega}$ vs $C_{l\beta,\Omega}$ for a first-order response should be a straight line with a (negative) slope of $-\mathrm{T}_{\mathrm{sep}}$ and an extrapolated intercept with the x axis, i.e., $C_{l\dot{\beta},\Omega} = 0$, at $C_{l\beta,\mathrm{att}}$.

Figure 3 shows such a cross plot for the data of Ref. 7. The success of the simple first-order lag model is remarkable, failing only at the highest angle of attack, where the deviation from a straight line at higher frequencies indicates a second- or higher-order lag. This may also be the case at lower angles of attack, but the corresponding time constants ($\propto$ slope) are much lower so that the oscillation frequencies do not approach the system natural frequency.

Attached flow static derivatives and separated flow time constants derived from using equation (21) are shown in Fig. 4. The steady-state $C_{l\beta}$ characteristic has been extended into the fully stalled incidence range using data from tests on a wing of similar planform and leading-edge profile.[21] The results are again most encouraging with $C_{l\beta,\mathrm{att}}$ approaching the steady-state $C_{l\beta}$ curve at low angles of attack, where the separated flow contribution to the aerodynamic loads is small, and scaling roughly with the lift coefficient C_L at intermediate angles of attack.[20,22] At high angles of attack, where the upper surface flow is fully separated, any rolling moment is almost entirely a result of the attached flow on the lower surface and, hence, $C_{l\beta,\mathrm{att}}$ approaches $C_{l\beta}$ once more. The effective time constant variation with angle of attack shows some scatter because of

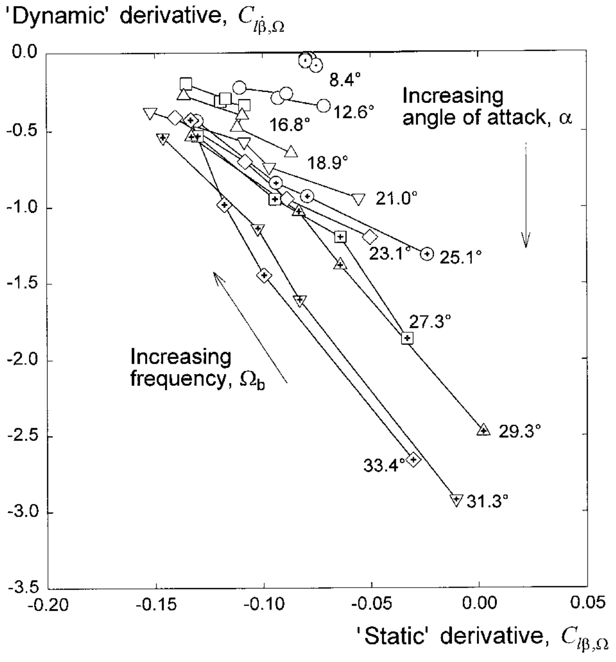

Fig. 3 Cross plot of dynamic and static rolling moment derivatives for a 60-deg delta wing.

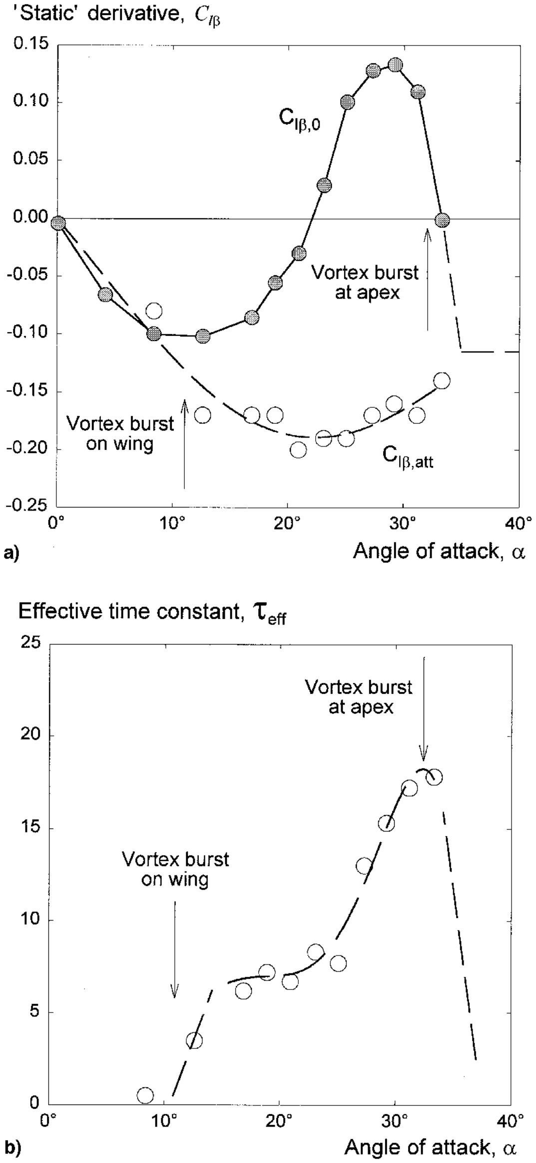

Fig. 4 Variation of attached flow static derivative and separated flow time constant with angle of attack for a 60-deg delta wing.

the limited number of frequencies tested, but clearly subdivides into two distinct regions that can conceptually be related to the wing flowfield.

1) $\alpha = 0 \rightarrow 12$ deg: Small separated leading-edge vortex flow contribution, with nondimensional time constants of the order of 1 (corresponding to typical response times of unburst vortices), but obscured by the scatter in the data.

2) $\alpha = 12 \rightarrow 32$ deg: Strong leading-edge vortex flow contribution, with vortex breakdown on the wing and time constants increasing from about 6 to about 18 as the angle of attack increases and the burst point moves up the wing toward the apex.[23]

Above $\alpha = 32$ deg the leading-edge vortex structure has subsided and the wing upper surface flow is essentially fully separated. The trends with angle of attack seen in Fig. 1, coupled with data from Ref. 24 and experience with unpublished combat aircraft testing at the Defence Evaluation and Research Agency (DERA) suggest that in this third region the associated time constants fall to a very low value, as the rolling moment is once more dominated by the attached lower surface flows.

It should be noted that there are considerable experimental differences between the static and dynamic tests reported in Ref. 7. Although the same wind tunnel and model were apparently used, the static tests used a conventional mechanical six-component balance with a single vertical strut support to measure forces for a sideslip range of ± 5 deg. Dynamic tests were conducted with the model mounted above a single horizontal strut oscillated laterally to give a sideslip range of ± 2 deg, while rolling and yawing moments were measured using a crude two-component strain-gauge balance at the model c.g. As result, inconsistencies between the static and dynamic measurements could arise from three sources: 1) Changes in support blockage, 2) nonlinearities because of variation in the sideslip range, and 3) possible normal force coupling on the simple two-component dynamic balance.

Comparison of Derivative and Transfer Function Models

The conventional aerodynamic or stability derivative model of Eqs. (6) and (7) can be rewritten as an equivalent transfer function

$$G_{\text{stab}}(\bar{s}) = C_{l\beta} + C_{l\dot{\beta}}\,\bar{s} \tag{22}$$

and similarly the aerodynamic transfer function of Fig. 2b is

$$G_{\text{atf}}(\bar{s}) = C_{l\beta,\text{att}} + \frac{C_{l\beta,\text{sep}}}{1 + T_{\text{sep}}\bar{s}} \tag{23}$$

For purposes of a preliminary comparison, consider representative values for a conventional stability derivative and an aerodynamic transfer function model for a combat aircraft configuration at an intermediate angle of attack. The steady-state derivatives are

$$C_{l\beta,0} = +0.3, \qquad C_{l\dot{\beta},0} = -2.2 \tag{24a}$$

i.e., statically unstable, whereas the equivalent attached and separated flow contributions are

$$C_{l\beta,\text{att}} = -0.25, \qquad T_{\text{sep}} = 4.0 \tag{24b}$$

and, hence, from Eq. (18b)

$$C_{l\beta,\text{sep}} = +0.55 \tag{24c}$$

Giving

$$G_{\text{stab}}(\bar{s}) = 0.30 - 2.2\bar{s} = 0.30(1.0 - 7.33\bar{s}) \tag{25a}$$

$$G_{\text{atf}}(\bar{s}) = -0.25 + \frac{0.55}{1 + 4.0\bar{s}} = 0.30\left(\frac{1 - 3.33\bar{s}}{1 + 4.0\bar{s}}\right) \tag{25b}$$

The frequency responses of Eqs. (25) are compared in Fig. 5. Clearly, there are significant differences: Both models give similar phase lags at low frequencies, but the amplitude ratio trends are in opposite directions, with the stability derivative model giving a continuously increasing amplitude ratio with frequency. Further, for the stability derivative model, the phase lag is determined by the value of the dynamic derivative extrapolated to zero frequency, $C_{l\dot{\beta},0}$. Consequently, if, as is common practice, a value of $C_{l\dot{\beta},\Omega}$ at a representative frequency Ω is used instead, a considerable error could be introduced. This is of particular concern at higher angles of attack where $C_{l\dot{\beta},\Omega}$ can change sign as frequency is increased.

This example demonstrates the shortcomings of the conventional stability derivative model, particularly for rapid aircraft motions, for example, the response to a rudder doublet input, at higher angles of attack where the separated flow response time exceeds the duration of the disturbance and, hence, the initial transient behavior predominates.

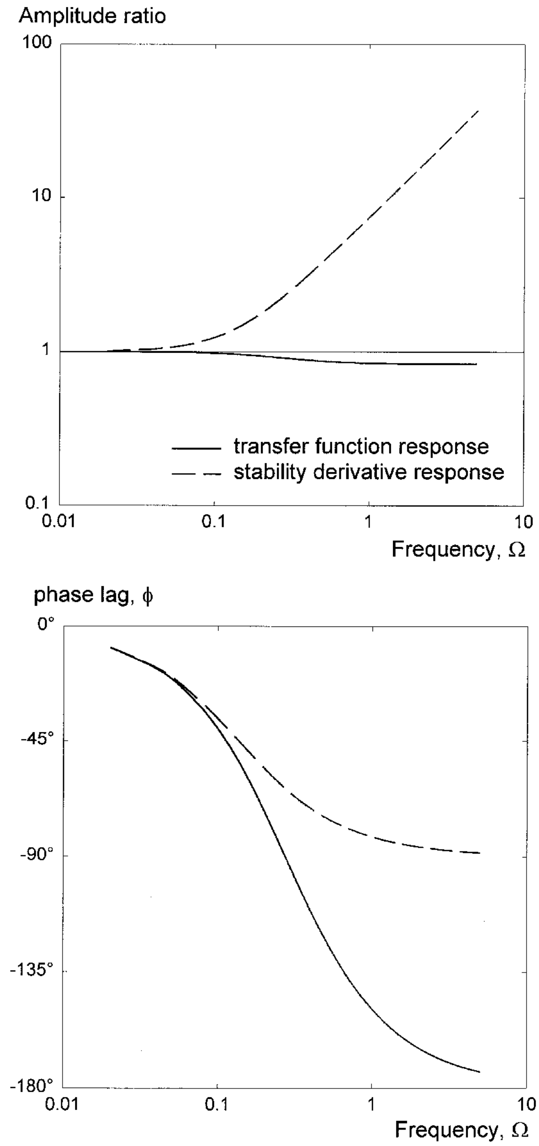

Fig. 5 Comparison of stability derivative and aerodynamic transfer function frequency responses.

Application and Extension

It is clear that experimentally observed variations in static and dynamic derivatives with (small-amplitude) oscillation frequency are consistent with an aerodynamic transfer function model. It is equally clear that for successful identification of a model a number of factors are important.

1) A physically meaningful structure for the model: For example, it may be necessary to bring in additional terms to differentiate between unburst and burst vortex time scales (Fig. 2c). A physically meaningful structure is essential for proper application of corrections for Reynolds number, Mach number, and wind-tunnel interference, and for the rapid assessment of effects of configuration changes.

2) Consistency between steady-state and oscillatory measurements: Ideally, the same experimental apparatus should be used for both measurements. Particular attention must be paid to relative amplitudes, e.g., $\Delta\beta_{\text{static}}$ vs $\Delta\beta_{\text{dynamic}}$, in regions where the steady-state aerodynamic characteristics are strongly nonlinear. This will act as an additional limit on the test envelope of translational, e.g., plunge or sway, rigs, where the

aerodynamic motion amplitude [Eq. (5)] varies with the oscillation frequency and freestream velocity.

3) Adequate frequency range: Given the number of possible model parameters, a wider frequency range with closer-spaced data points than conventionally used is essential. High-frequency data assists in identification of the order of the aerodynamic response and in determination of the attached flow contribution(s). At the lower end, experience has shown that dynamic derivatives in particular can change very rapidly with frequency. However, care must be taken at higher oscillation frequencies to account for sting and model deflections, whereas the signal-to-noise ratio can become a problem at lower frequencies.

Given these prerequisites, a transfer function representation of the aerodynamic stability characteristics has a number of advantages: 1) Readily integrated into existing aircraft simulation codes, consistent methodology; 2) valid for arbitrary (small-amplitude) aircraft motion profiles; 3) wide range of existing linear system analysis techniques can be applied; 4) flexibility in adapting to configuration changes, reduced test requirements; 5) aids application of experimental corrections; and 6) assists in identification of underlying flow physics.

The first characteristic is particularly significant, and in the author's opinion justifies the use of transfer function-based methodologies on its own. The application of common modeling methodologies would facilitate closer cooperation and interaction between unsteady aerodynamicists on one hand and the flight vehicle simulation community on the other.

A preliminary step in the application of the ATF modeling methodology is the direct replacement of the current aerodynamic derivative formulation with individual linear transfer functions relating each of the six aerodynamic force and moment coefficients (C_x, C_y, C_z, C_l, C_m, and C_n) to each of the five translational and rotational velocities (α, β, p, q, and r). This model rests on the same assumption as the aerodynamic derivative model of locally linear, small-amplitude motion, but incorporates the time dependency of the aerodynamic responses properly. The steady-state gains, e.g., $C_{l\beta,\mathrm{att}}$ and $C_{l\beta,\mathrm{sep}}$, and time constants, e.g., T_{sep}, are functions of α and β. Also implicit in this model is the assumption that the aerodynamic loads can be separated into orthogonal components, the time-dependent responses determined individually, and the resulting time histories then recombined. This will only be the case if motion rates or amplitudes are low enough for linear superposition to apply.

When considering further developments or extensions of a modeling technique it is important not to lose sight of the application requirements. The primary objective of dynamic wind-tunnel testing is to provide data for prediction and analysis of aircraft flight dynamics, both off-line and in piloted simulations. As a result, Bryan's formulation of the conventional linearized stability derivative model in 1911 was driven by the need for an analytical determination of an aircraft's stability.[1] Eighty years later, the aerodynamic transfer function model is ideally suited for integration into numerical methods for the simulation and assessment of handling characteristics and maneuverability, using the wide range of modeling and analysis methods developed in the field of control systems engineering.

Conclusions

It is now widely acknowledged that the conventional stability derivative model for representation of aerodynamic loads in the equations of motion is no longer adequate for modern combat aircraft maneuvering capabilities. An examination of published small-amplitude oscillatory test data at higher angles of attack has shown the presence of significant motion frequency effects on the static and dynamic derivatives that cannot be reconciled with the conventional stability derivative model structure.

For small-amplitude (but not necessarily low-rate) aircraft motions, the time-dependent response of the aerodynamic loads (including the frequency effects referred to earlier) can be adequately modeled by a linear aerodynamic transfer function. The following items are crucial to the success of this model.

1) A physically meaningful structure for the model, based on an understanding of the underlying fluid mechanics.

2) Consistency between steady-state and oscillatory test conditions, in particular, support interference levels and motion amplitudes.

3) Adequate frequency range for oscillatory testing.

Acknowledgment

This work was supported by the Ministry of Defence under the Applied Research Programme.

References

[1] Bryan, G. H., *Stability in Aviation*, Macmillan, London, 1911.

[2] Etkin, B., *Dynamics of Flight*, Wiley, New York, 1959.

[3] Hancock, G. J., *An Introduction to the Flight Dynamics of Rigid Aeroplanes*, Ellis Horwood, New York, 1995.

[4] Hanff, E. S., "Direct Forced-Oscillation Techniques for the Determination of Stability Derivatives in Wind Tunnels," *Dynamic Stability Parameters*, AGARD LS-114, March 1981 (Lecture 4).

[5] Campbell, J. P., and Woodling, C. H., "Calculated Effects of the Lateral Acceleration Derivatives on the Dynamic Lateral Stability of a Delta-Wing Airplane," NACA RM L54K26, Jan. 1955.

[6] Nguyen, L. T., "Evaluation of Importance of Lateral Acceleration Derivatives in Extraction of Lateral-Directional Derivatives at High Angles of Attack," NASA TN D-7739, Oct. 1974.

[7] Lichenstein, J. H., and Williams, J. L., "Effect of Frequency of Sideslipping Motion on the Lateral Stability Derivatives of a Typical Delta-Wing Airplane," NACA RM L57F07, Sept. 1957.

[8] Fisher, L. R., "Experimental Determination of the Effects of Frequency and Amplitude of Oscillation on the Roll-Stability Derivatives for a 60° Delta-Wing Airplane Model," NASA TN D-232, March 1960.

[9] Lichenstein, J. H., and Williams, J. L., "Low-Speed Investigation of the Effects of Frequency and Amplitude of Oscillation in Sideslip on the Lateral Stability Derivatives of a 60° Delta-Wing, a 45° Swept-back Wing and an Unswept Wing," NASA TN D-896, May 1961.

[10] Letko, W., "Experimental Determination at Subsonic Speeds of the Oscillatory and Static Lateral Stability Derivatives of a Series of Delta Wings with Leading-Edge Sweep from 30° to 86.5°," NACA RM L57A30, April 1957.

[11] Fletcher, H. S., "Low-Speed Experimental Determination of the Effects of Leading-Edge Radius and Profile Thickness on Static and Oscillatory Lateral Stability Derivatives for a Delta Wing with 60° of Leading-Edge Sweep," NACA TN 4341, July 1958.

[12] Riley, D. R., Bird, J. D., and Fisher, L. R., "Experimental Determination of the Aerodynamic Derivatives Arising from Acceleration in Sideslip for a Triangular, a Swept and an Unswept Wing," NACA RM L55A07, March 1955.

[13] Coe, P. L., Graham, A. B., and Chambers, J. R., "Summary of Information on Low-Speed Lateral-Directional Derivatives Due to Rate of Change of Sideslip," NASA TN D-7972, Sept. 1975.

[14] Atkinson, P., *Feedback Control Theory for Engineers*, Heinemann Educational Books, Ltd., London, 1972.

[15] Etkin, B. H., "Aerodynamic Transfer Functions: An Improvement on Stability Derivatives For Unsteady Flight," Univ. of Toronto, UTIA Rept. 42, Toronto, ON, Canada, 1956.

[16] Tobak, M., and Schiff, L. B., "Aerodynamic Mathematical Modelling—Basic Concepts," *Dynamic Stability Parameters*, AGARD LS-114, March 1981 (Lecture 1).

[17] Chin, S., and Lan, C. E., "Fourier Functional Analysis for Unsteady Aerodynamic Modelling," *AIAA Journal*, Vol. 30, No. 9, 1992, pp. 2259–2266.

[18] Marques, F., and Anderson, J., "Modeling and Prediction of Non-Linear Unsteady Aerodynamic Loads by Adaptive Neural Networks," *Unsteady Aerodynamics*, Royal Aeronautical Society Conf., London, July 1996 (Paper 4).

[19] Coe, P. L., Graham, A. B., and Chambers, J. R., "Summary of Information on Low-Speed Lateral-Directional Derivatives Due to Rate of Change of Sideslip," NASA TN D-7972, Sept. 1975.

[20]Campbell, J. P., Johnson, J. L., and Hewes, D. E., "Low-Speed Study of the Effect of Frequency on the Stability Derivatives of Wings Oscillating in Yaw with Particular Reference to High Angle-of-Attack Conditions," NACA RM L55H05, Nov. 1955.

[21]Greenwell, D. I., "Control of Asymmetric Vortical Flow over a Delta Wing at High Angles of Attack," Ph.D. Dissertation, Univ. of Bath, Bath, England, UK, 1993.

[22]Shanks, R. E., "Low-Subsonic Measurements of Static and Dynamic Stability Derivatives of Six Flat-Plate Wings Having Leading-Edge Sweep Angles of 70° to 84°," NASA TN D-1822, July 1963.

[23]Wentz, W. H., and Kohlman, D. L., "Vortex Breakdown on Slender Sharp-Edged Wings," *Journal of Aircraft*, Vol. 8, No. 3, 1971, pp. 156–161.

[24]Myatt, J. H., "Modelling the Rolling Moment on the 65° Delta Wing for Rolling Motions at High Angle of Attack," Ph.D. Dissertation, Stanford Univ., Stanford, CA, April 1997.